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**TAIL RISK IN THE AUTOMATED CLEARING
SETTLEMENT SYSTEM (ACSS)**

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Tail Risk in the Automated Clearing Settlement System (ACSS)

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TABLE OF CONTENTS

Executive Summary	4
1. Introduction.....	5
2. Data.....	8
3. Methodology	9
3a. Block Maxima	
3b. Peak-over-Threshold	
3c. Model Diagnostics and Robustness	
3d. Estimating Extreme Quantiles	
4. Results.....	13
4b. A Dynamic Collateral Pool and Individual Participant Contributions	
4. Conclusion.....	15
References	17
Tables and Figures.....	18

Executive Summary

We investigate a framework to inform measurement of credit exposure in the Automated Clearing Settlement System (ACSS), which contributes to improved management of credit risk. Following Arjani (2016), we propose an empirical methodology based on extreme-value theory to assess credit exposure under extreme tail events. Applying the methodology to ACSS data, we obtain estimates for a shape parameter that governs the tail distribution of exposures. We then discuss the reliability of our estimates and examine how a collateral pool derived from our model estimates evolves over time. Our findings have policy implications for both settlement and risk model determination in payment clearing and settlement systems.

Key messages are as follows.

- Direct Clearers (DCs) are exposed to credit risk in the ACSS. This risk materializes in the event of DC default, via a survivors-pay loss-allocation mechanism in the ACSS.
- In this paper, an extreme-value methodology is applied to obtain out-of-sample forecasts of potential credit exposures that could emerge for surviving DCs in a default scenario under unusual, tail-risk conditions. This is in contrast to more conventional approaches, including historical Value-at-Risk (VaR) models that draw inference strictly from past experience which could be benign and so underestimate potential tail risk.
- Based on daily ACSS batch-entry data spanning 15 years, we estimate credit exposures which could emerge in extreme tail events, such as those that might arise once in 50 years. Such model-based quantitative estimates provide additional understanding of payments system data and can inform both the ongoing monitoring of payment flows and policy decisions about system risk controls, such as required collateralization.
- A key question for practitioners and policymakers is the extent to which system risk controls – and in particular collateralization of credit exposure – should focus on measures of extreme tail risk such as those explored in this paper, or on more conventional measures that are based only on historical experience.

1. Introduction

This paper explores the measurement of credit risk exposure in the Automated Clearing Settlement System (ACSS). The ACSS facilitates the clearing and settlement of primarily low-value electronic and paper-based payment items on behalf of Canadian financial institutions and their customers. In 2016, the ACSS cleared over \$6.6 trillion in value, representing roughly 7.4 billion individual payments. These payments underpin much of the day-to-day activity in the Canadian economy.

To economize on costs related to, for example, information technology requirements, and reflecting the predominantly paper-based payments landscape that existed in Canada at the time of its introduction in 1984, the ACSS is designed as a debit-entry batch-total clearing system. ACSS Direct Clearers (DCs) determine total payment volume and value information from payment files/items exchanged during the day with other financial institutions, and subsequently make bilateral entries into the ACSS against each other based on these amounts. Each day gives rise to multiple payment files/items being exchanged between financial institutions in Canada – reflecting different payment instruments, alternative exchange windows, etc. – which, in turn, can generate multiple clearing entries by DCs to the ACSS.

Based on the bilateral entries by DCs during a given day, the ACSS performs ‘clearing’ by calculating each DC’s final multilateral net position, i.e., whether it is in a net debit or net credit position. Specifically, this is a single balance for settlement generated by the ACSS for each DC – which could be positive (i.e., net credit, the DC is owed funds by the system to complete ACSS settlement), negative (i.e., net debit, the DC owes funds to the system to complete ACSS settlement), or zero. It follows that final net positions of DCs in the ACSS are summary measures, resulting from roughly 27 million unique daily transactions. Settlement of these final net positions takes place on the next business day by transfer of corresponding amounts across DCs’ ACSS settlement accounts held at the central bank. The Large-Value Transfer System (LVTS) is the mechanism used to transfer funds to and from DCs’ settlement accounts at the central bank. The ACSS is commonly referred to as a deferred net settlement system; as indicated above, DCs’ final positions are settled on a multilateral net basis on the following business day.¹ Multilateral netting employed by the ACSS helps reduce cost and lowers settlement risk exposure between DCs.

While settlement (credit) risk exposure is reduced between DCs as a result of multilateral netting, it is not necessarily eliminated. By design, DCs are exposed to credit risk vis-à-vis each other for the period between when an entry is made to the ACSS and when final settlement occurs the following business day.² Moreover, as a

¹ See Olivares and Tompkins (2016) for a global review of payment systems. Examples include the Large Value Transfer System (LVTS) in Canada, Federal Reserve Wire Network (Fedwire) in the United States, and Clearing House Automated Payment System (CHAPS) in the United Kingdom.

² Arjani (2016) provides a detailed analysis of the ACSS, including credit risk in that system.

debit-entry clearing mechanism (i.e., the bilateral ACSS entry is made by the DC that is owed funds following from payment exchange), there is no practical way by which a DC can limit its credit exposure each day. Put simply, an entry by a DC into the ACSS exposes it to credit risk vis-à-vis the receiving DC, and DCs are typically constrained in how much control they have over batch entries to the ACSS each day (which are driven largely by economic activities of clients).

Historically, as regards the management of credit exposure between DCs, and in the absence of requiring DCs to pledge financial collateral to secure this exposure, the ACSS has relied on relatively strict access criteria and, to a lesser extent, cross-system liquidity swaps against DCs' LVTS positions to manage exposures.³ Despite having these measures in place, in the event that a DC with a final multilateral net debit position is unable to meet its ACSS settlement obligation via payment to the central bank, a legally binding loss-sharing agreement requires surviving DCs to collectively cover any shortfall resulting from the defaulter's obligation to the system.⁴

In 2016, the Bank of Canada designated the ACSS as a "prominent payment system", which essentially means that the central bank considers that the ACSS poses sufficient risk to the Canadian financial system and economy that it should be held to enhanced oversight scrutiny. From the perspective of credit risk management, most notably this means that ACSS DCs must now collectively post sufficient collateral to the system to cover, with a high degree of confidence, the credit exposure related to the default of the DC that would generate the largest single credit exposure in the system, under extreme but plausible market conditions.⁵ A pre-funded collateral pool made up of individual contributions from each DC would provide the needed resources to enable timely settlement in the event of default.

In this paper, we employ extreme-value empirical methods to inform the potential magnitude of credit exposure that could arise in the ACSS, which contributes to improved management of credit risk in the ACSS, and may also inform determination of an appropriate collateral pool.

A conventional approach to collateralization sets a collateral pool with reference to a point in the distribution of potential losses observed over a recent reference period. For example, using a Value-at-Risk (VAR) method to determine a collateral pool for ACSS could require a pool sufficient to cover a target VAR threshold, such as the largest or

³ These swap arrangements are known as 'Settlement Exchange Transactions', or SETs, and are available to financial institutions that are both LVTS Participants and ACSS DCs. A fundamental purpose of SETs is to reduce overnight interest costs that could accrue from large dislocations between settlement positions in the ACSS and LVTS.

⁴ It follows that intraday liquidity capacity in the LVTS is directly related to prospective materialization of credit risk in the ACSS. Of note, there has been no default in the ACSS since its inception in 1984. ACSS DCs are regulated deposit taking institutions in Canada.

⁵ See <http://www.bankofcanada.ca/wp-content/uploads/2016/02/criteria-risk-management-standards.pdf>

99th quantile of the net debit positions observed during the last 12 months. (See Hendricks 1996 for a comprehensive discussion of VaR measures.) A shortcoming of VaR, however, is that it does not inform on potential losses beyond the specified loss threshold (Gregory 2015) – which could vary in magnitude depending on the degree of heaviness (or fatness) characterizing the tail of the distribution.

Extreme-value theory (EVT) uses available historical data and statistical theory to approximate (estimate) the shape of the tail distribution to inform likelihood of extreme events occurring over long time horizons. We explore and apply these methods to our data in this paper.⁶ Put briefly, EVT derives from an analog to the Central Limit Theorem, where a series of maximum values from independently drawn random samples will (asymptotically) converge to a given theoretical distribution governed by key parameters. Using our available historical series of tail, or outlier, ACSS net debit positions as an estimation sample, we solve for the theoretical distribution's parameters that best explain the shape of the observed empirical tail distribution. Subsequently, we can project the empirical tail distribution to obtain corresponding theoretical exposure threshold levels and their probabilities. As a result, the methodology uncovers estimates for tail quantiles, or extreme events, not directly observable in recent data – such as one-in-50 year events.

The reliability of such a projection is, of course, conditional on the estimation sample at hand and therefore entails model risk. Intuitively, our model estimates are informative to the extent that historical data contain a sufficient sample of tail events.

As a preview of our results, our benchmark set of estimates suggests that the tail distribution is governed by a shape parameter that centers around zero, which implies that the distribution of ACSS credit exposure is not heavy tailed. However, our analysis also highlights the sensitivity of our estimates to model uncertainty. Finally, we illustrate the application of extreme value methods to determine a collateral pool with respective DC allocations updated using monthly data.

The next two sections of the paper discuss data and methodology. Section 4 presents results, and Section 5 concludes.

⁶ Generally speaking, EVT refers to the science of estimating the tails of a distribution. Longin (2016) reviews a comprehensive collection of papers about EVT and its applications in finance. Galbraith and Zernov (2009) study extreme dependence in financial returns. Danielsson and Zhou (2016) study and compare estimation uncertainty in VAR and expected shortfall analysis. Hull (2007) discusses the benefits of expected shortfall analysis compared to VAR for portfolio management.

2. Data

The ACSS clears the majority of non-cash retail payments in Canada, about 27 million transactions each day worth on average \$25 billion. These are payments where a transfer of funds between accounts held at different financial institutions – which are represented by different DCs in the ACSS – is required. In 2016, the ACSS cleared over \$6.6 trillion in value, representing roughly 7.4 billion individual payments. Figure 1 plots aggregate monthly values and volumes of payments cleared through ACSS from January 2002 to March 2017, corresponding to the historical period used in our estimation sample.

As noted above, the ACSS produces a final settlement balance for each DC – its multilateral net position – for a given payments cycle per business day. Figure 2 presents incoming and outgoing payment flows for a selected DC and the resulting multilateral net position over time. Credit risk materializes when a DC is incapable of meeting its final settlement obligation (a debit, or payable, multilateral net position) – that is, at time of settlement it is unable to deliver funds to its ACSS settlement account at the Bank of Canada in an amount equivalent to its final multilateral net debit position.

The historical empirical distribution of daily multilateral net debit positions serves as a natural starting point to study exposures that could arise in ACSS. Figure 3 plots the densities of historical net debit positions for selected DCs from our sample period. The densities are highly skewed with critical mass centering near the origin. Figure 4 plots the density of the daily maximum net debit position observed in the ACSS across all DCs. While the mass of observations is below \$500 million, the density is again highly skewed, with a small number of observations between one and two billion dollars. The highly skewed nature of the data raises questions about whether large outliers may be underrepresented in the sample.

Figure 5 illustrates the effectiveness of a collateral pool calibrated from historical empirical quantiles *ex post*. For instance, the 99th empirical quantile is expected to give rise to a shortfall in one percent of cases, or once in every 100 business days. Understanding the potential magnitude of a shortfall, where a net debit position exceeds the available collateral pool, is an important component for informing risk tolerance. These perspectives motivate the use of an extreme-value statistical model to shed light on the underlying uncertainty of tail risk, and to inform the risk versus efficiency trade-off when choosing a given collateral pool.

3. Methodology

There are two main approaches to modeling the tails of a distribution: block maxima and peak over threshold. The peak-over-threshold approach models the empirical tail above a threshold level while the block maxima approach models the maximum values from a specified window of time, or block, such as weekly or monthly intervals. We provide an overview of these methods below.

(a) Block Maxima Method

For this method, the data are grouped into sequences of observations of a predetermined length (blocks) from which the maximum value of each block is selected. This, in turn, generates a series of block maxima. It can be shown that the distribution of block maxima, rescaled using location and shape parameters, converges asymptotically to a generalized extreme value (GEV) distribution. This limiting distribution corresponds to one of three types of extreme value distributions, regardless of the underlying distribution of the population (Coles 2001). Formally, we define the series of maxima as $M_m = \max\{X_1, X_2, \dots, X_m\}$, where the set of independent random variables X_1, X_2, \dots, X_m have a common, but unknown, distribution, F . In the current context, X_i are daily net debit positions of participants observed over m periods based on daily, weekly and monthly sized blocks.

Specifically, the class of limiting distributions have the form:

$$G(z) = \exp\left(-\left(1 + \xi\left(\frac{z - \mu}{\sigma}\right)\right)^{-1/\xi}\right)$$

which incorporates three types of extreme value distributions known as Gumbel, Frechet and Weibull distributions and where z refers to the series of maxima net debit positions (Fisher-Tippett Theorem). The parameters that define the family of these distributions, and which are to be estimated using historical data, are a location parameter μ , a positive scale parameter σ , and a shape parameter ξ .

The shape parameter ξ determines the behavior of the tail distribution; that is, ξ indicates which family of extreme value distributions correspond to the data. Therefore, ξ is of central interest in this analysis. Specifically, an estimated shape parameter that is negative, $\xi < 0$, corresponds to a Weibull distribution that is theoretically bounded. In contrast, a zero shape parameter corresponds to an unbounded but well-behaved distribution (Gumbel) which includes the Normal. A positive shape parameter is associated with an unbounded but heavy tailed distribution (Frechet). Estimating ξ reveals the tail behaviour of the data, and the strength of that categorization depends on the degree of confidence in the estimation. As described in Coles (2001), through inference on ξ , the data themselves determine the most

appropriate way to summarize tail behaviour. Uncertainty in the inferred value of ξ therefore implies lack of certainty as to which of the three distributions best characterizes the data.

(b) Peak-over-Threshold Method

One potential drawback of the block maxima approach is that some blocks may contain more extreme observations than others and these values are excluded from the estimation. As a result, block maxima does not fully use the information content when data are available at higher frequency. The peak-over-threshold approach builds on the GEV distribution by modeling a conditional series of tail exposures above a given threshold. As a result, there is a direct relationship between the two approaches.

Specifically, we define the subset of observations E_t that exceed a given high threshold τ such that $E_t = X_t - \tau$ where an individual observation is defined as $e = x - \tau$, for $x \geq \tau$. The conditional distribution of the excesses E_t can be approximated by the Generalized Pareto Distribution (GPD) if the threshold τ is sufficiently high and some regularity conditions hold (Balkema-de Haan-Pickands Theorem). We can write the limiting distribution as:

$$H(E) = 1 - \left(1 + \frac{\xi e}{\tilde{\sigma}}\right)^{-1/\xi}$$

where $\tilde{\sigma}$ and ξ are the scale and the shape parameters, respectively, and where $e > 0$, $e \leq -\frac{\tilde{\sigma}}{\xi}$, and $\tilde{\sigma} = \sigma + \xi(\tau - \mu)$. Similarly, the shape parameter defines the behaviour of the tail.

(c) Model Diagnostics and Robustness

We use the method of maximum likelihood to obtain estimates of the model parameters using the algorithm developed by Roodman (2015). The principle of maximum likelihood estimation is to identify the model parameter values that have the greatest probability of generating the observed data. In some cases, the (log) likelihood function has an explicit functional form that can be solved analytically. But, in most applications, an analytical solution is not possible and numerical methods are required. Model diagnostics entail comparing the empirical and fitted (estimated) distribution functions. Furthermore, stability of parameter estimates across both methods and over a range of model choices serve as additional robustness checks.

While assessing model fit is important, a key consideration for ascertaining the reliability of our estimates lies in the implicit assumption that the asymptotic properties under which the model is based holds. For instance, selecting an estimation sample of tail observations drawn from a short window (under the GEV approach) or a low threshold (under the GPD approach) is likely to lead to a biased model based on a contaminated estimation sample that incorrectly includes some observations, but

where model parameters will be estimated with greater precision (low variance) given the larger sample size. In contrast, a longer window, such as annual, or a high threshold choice will correspond to unbiased model estimates but which are imprecisely estimated due to a smaller sample size. To illustrate, a series of annual maxima (high threshold) might best satisfy the asymptotic criteria of the model but our estimation sample would include (only) 15 observations. As a result of these tradeoffs, we show a range of parameter estimates and highlight some of the uncertainty embedded in our modeling assumptions. The bootstrap bias correction estimator of Giles, Feng and Godwin (2013) minimizes some of the small sample bias.

A second implicit assumption requires the series of maxima to be independent and identically distributed (iid). An independence assumption for the series of maxima is likely to be a reasonable approximation so long as the block length (and corresponding threshold choice) is sufficiently large. For instance, it is unlikely that there is any dependence between observations from an annual series of maxima as compared to a shorter block such as daily or weekly. Consequently, the choice of block length and threshold level also influence the degree to which the independence assumption holds. The block maxima approach is more robust to the iid assumption because the block allows for potential correlation (or clustering) within a specific block.

(d) Estimating Extreme Quantiles

After fitting the data to either GPD or GEV distributions, estimates of extreme quantiles can be obtained by inverting the distribution’s functional form and substituting the model parameters with the estimated ones. Using notation as in Coles (2001), we define $G(z_p) = 1 - p$, where z_p is the return level, or dollar value, associated with return period $1/p$. The level z_p is expected to be exceeded on average once every $1/p$ days. In the case of the 99th quantile, p is set to 0.01, corresponding to a once in every 100 days event. The return levels can then be plotted against their corresponding return periods resulting in a return plot, or risk curve. When the estimated shape parameter is negative, the risk curve will be convex and bounded implying an upper limit exists on tail outcomes.

In the case of GEV, estimates of extreme quantiles, z_p , are obtained by inverting $G(Z)$:

$$z_p = \mu - \frac{\sigma}{\xi} [1 - \{-\log(1 - p)\}^{-\xi}]$$

and in the case when using the GPD, inverting the limiting distribution $H(E)$ we obtain:

$$x_m = \tau + \sigma [(m\zeta_\tau)^\xi - 1],$$

where x_m denotes the level that is exceeded on average every m observations, ζ_τ denotes the proportion of the sample that exceeds the threshold level τ which measures the probability that an observation will exceed the threshold level.

4. Results

Table 1 presents the regression results from both GPD (Peak over Threshold) and GEV (Block Maxima) models. The GPD model is estimated using a threshold of \$800 million and consequently the estimation sample is based on 212 observations. For the GEV model, we show results for daily, weekly and monthly windows. The GPD threshold model and the GEV monthly window model parameter estimates are roughly equivalent.

A key finding from the set of regressions is that the estimates for the shape parameter are never statistically negative, indicating that the distribution of underlying extreme observations is unbounded. For robustness, we re-estimate all models using a bias correction estimator as in Giles, Feng and Godwin (2013) and Cox and Snell (1968). We observe that the standard errors decrease particularly for the monthly window results given their lower sample size, but in general the bias correction estimator does not change the statistical significance of any parameters. The last row of Table 1 provides return level estimates for one-in-ten year return periods and their respective confidence intervals for all models.

In the case of both GPD and monthly GEV regressions, the shape parameter is not statistically different from zero; suggesting that the distribution of extreme, or outlier, values is unbounded but not heavy tailed. In contrast, the opposite is found under the GEV daily and weekly window regressions, where the shape parameter is statistically positive suggesting a heavy tail distribution. An increasingly positive shape parameter implies a more skewed and heavy tailed distribution. Figure 6 plots a corresponding risk curve using the GPD model, mapping each quantile (or return period), measured in days, relative to its respective threshold exposure level (return level). As a result, Figure 6 visualizes the tradeoff when choosing a given collateral level in terms of efficiency and risk. Figure 7 provides a diagnostic plot comparing modeled and empirical quantiles; a straight line suggests a good fit.

As discussed in the methodology section, model estimates derived from smaller block sizes and lower thresholds are likely to be biased as the estimation sample departs from the asymptotic basis of the model. For instance, because a set of smaller outliers are included in the daily and weekly window analyses, they are potentially less informative as they shift the distribution of outliers towards the smaller ones. As we move from the daily window (unit of observation) towards the weekly window, the skewness becomes less pronounced. Furthermore, we observe a poor model fit for the daily and weekly GEV models as the theoretical quantiles match only the lower end of the tail. Consequently, the usual bias variance tradeoff ensues (as the daily and weekly regressions benefit from a larger estimation sample size). The choice of window length is similar to the choice of bandwidth in non-parametric statistics: in the limit, when a daily window is selected overlapping with the actual data, the empirical distribution converges to its original skewed form (see Figure 4).

A similar tradeoff follows in the case of GPD; Figure 8 shows varying shape parameters estimates as we adjust the threshold level from \$500M to \$1.4B. We observe that the shape parameter is initially slightly positive, then hovers around 0, and finally begins to increase as the threshold surpasses the \$1B level, at which point the estimation sample declines below 100 observations. By construction, the log likelihood measure improves as fewer observations imply less variability in the data and as a result does not provide a useful information criterion for choosing block size or threshold choice. For sensitivity analysis, Figure 9 provides a range of exposure (return) levels for a one-ten year return period for a range of (assumed) shape parameters.

A second source of model uncertainty is driven by whether the underlying distribution generating maxima is stationary. One approach to account for non-stationarity is to incorporate functions of time or other covariates for the model parameters. For instance, the data suggest a structural shift has occurred post-2007. To test this hypothesis, we run the GPD model splitting the sample into two periods, pre- and post-2007; we find an upward shift in the risk curve occurred following 2007. One factor that might explain this shift is the continued increase in volumes and values that clear through the ACSS, particularly in recent years. Figure 10 displays the increase in the 95th, 97th, and 99th model quantiles as the historical sample has increased over time. Both empirical and estimated model quantiles have remained approximately stable since 2012. In practice, as time evolves and the estimation sample increases, estimated parameters will change due to sampling variability but also potentially due to structural shifts that affect the underlying distribution generating tail outcomes. In other words, the shape of the tail distribution can change over time.

4b. A Dynamic Collateral Pool and Individual Participant Contributions

In this section, we illustrate the efficacy of a collateral pool derived from our model estimates, where the estimation sample is updated monthly to reflect incoming data. Data from 2002 to 2006 are used to initialize the model estimates for the first month of January 2007 and the model is updated monthly through to March 2017 (roughly ten years). Figure 11 displays the evolution of collateral pools based on 100 days, and 10 year, 20 year, and 50 year return periods using the GEV (monthly window) model. The distance between different return levels is governed by the shape parameter. For comparison, the figure also shows the evolution of the 99th empirical quantile. We find that a collateral pool based on the 10 year return period using the GEV model (of approximately two billion dollars) exhibits a shortfall in one month over the ten year period, of roughly \$30 million. In contrast, the 99th empirical quantile of roughly \$1 billion exhibits shortfalls roughly twice per year (ranging in magnitudes).

While the estimation sample to obtain a system-level cover-one collateral pool is derived from the set of largest net debit positions observed in ACSS each day, the model can also be applied separately for each DC's respective historical set of end-of-day net positions. Such individual participant model estimates for tail risk could inform collateral pool allocation criteria. For example, each DC could contribute collateral

proportional to the model's estimate of a tail quantile for that participant, such as a one-in-ten-year event. Figure 12 illustrates each DC's contribution under such an approach at a snapshot in time (using data up until a given period). Based on this approach and similarly updating individual participant estimates monthly, Figure 13 depicts the aggregate collateral pool and respective DC contributions updated monthly from 2007 to 2017.

Interestingly, our analysis highlights important heterogeneity across DCs in terms of the shape parameter governing the tail distribution of their net debit positions and how these estimates can change over time; see Figure 14. Individual model estimates at the DC level can also serve as a monitoring and anomaly detection tool as well as identify changes in risk that specific DCs bring to the system. Importantly, the shape of the tail distribution of the daily maximum of net debit positions (which is the focus of this paper) differs from individual DC tail distributions. As a result, an alternative approach to derive a model-based collateral pool might be to estimate models for DCs separately and base the system-wide collateral pool according to the DC that poses the highest risk (or largest tail exposure). Future research might investigate tail dependence between select DCs.

5. Conclusion

Given recent interest in collateralizing exposures in the ACSS, this paper proposes a framework for measuring credit exposure in the ACSS using extreme-value empirical methods. We review our set of findings below.

First, we estimate a series of models and conduct small sample bias tests using the extreme occurrences of historical end-of-day ACSS net debit positions and uncover the key parameters that categorize best-fitting GEV and GPD distributions. From the estimated model parameters, we obtain a risk curve that quantifies estimated exposure size and cadence of such exposure (Figure 6). In our benchmark models (GEV monthly window and GPD with an \$800M threshold), the shape parameter is found to be statistically no different than zero implying an unbounded, but not heavy (or fat), tail of potential losses or risk curve.

Second, we illustrate how structural changes in the payments environment can affect the degree of risk, or model parameter estimates, observed in the data. We find significant differences when estimating our model using data from two periods, pre- and post-2007, where the latter is categorized by increasing volumes as well as other changes stemming from the post financial crisis era. Similarly, financial transactions such as SETs can also affect the tail distribution. As a result, a key theme of our paper surrounds the robustness of our model estimates and the importance of sensitivity analysis. Other risk management methods, such as stress testing, can also be used to complement statistical-based measures considered in this paper.

Finally, we illustrate how a collateral pool and individual DC contributions can be determined in practice from our benchmark models where estimates are updated on a monthly basis as new data arrives.

The extreme value methodology applied in this paper provides a useful tool for understanding tail risk and to inform system design. A key question for practitioners and policymakers is the extent to which risk controls and collateral requirements should focus on exposures in extreme tail events, as explored in this paper, or on more conventional models that are based only on historical experience. Since it is impossible for a DC in ACSS to constrain its exposures in that system (because it is a debit entry system), and since, as shown here, exposures can be large in tail events, consideration might be given to use of extreme value methods instead of conventional, historical VaR methods to inform collateral requirements. An important consideration in this regard is the tradeoff between collateral efficiency and risk exposure.

Given the modernization of Canada's payment systems currently underway, these various issues are central to system design considerations and policy rules governing the next-generation batch clearing and settlement system.

References

- Arjani, N. (2016) "A Primer on Credit Risk in Payments Canada's Automated Clearing Settlement System (ACSS)," Payments Canada Discussion paper No. 6.
- Coles, S. (2001) "An Introduction to Statistical Modeling of Extreme Values," Springer-Verlag, 1st Edition, London.
- Cox, D.R, and E.J. Snell. 1968. A general definition of residuals. *Journal of the Royal Statistical Society. Series B.*
- Danielsson, Jon and Zhou, Chen. (2016). "Why is Risk So Hard to Measure?" DNB Working Paper No. 494, January 2016.
- Galbraith, John and Zernov, Serguei (2009). "Extreme dependence in the NASDAQ and S&P 500 composite indexes," *Applied Financial Economics*, Vol. 19, No. 13.
- Giles, D. E., Feng, H., and Godwin, R. (2013) "On the Bias of the Maximum Likelihood Estimator for the Two-Parameter Lomax Distribution," *Communications in Statistics - Theory and Methods*, Vol. 42, No. 11, 1934-1950.
- Gregory, J. (2015) "Counterparty Credit Risk: The New Challenge for Global Financial Markets," *The Wiley Finance Series*, 2nd Edition, West Sussex.
- Hendricks, D. (1996) "Evaluation of Value-at-Risk Models Using Historical Data," *Economic Policy Review*, Vol.2, No. 1, 39-70
- Hull, John (2007). VAR versus Expected Shortfall, Risk.net article. Available at <https://www.risk.net/risk-magazine/technical-paper/1506669/var-versus-expected-shortfall>.
- Jacho-Chávez, D.T., Petrunia, R., Sabetti, L., and Voia M., 2018. "Tail Risk in a Retail Payments System," *Journal of Economics and Statistics*, De Gruyter, vol. 238(3-4), pages 353-369, July.
- Longin, F. (2016) "Extreme Events in Finance: A Handbook of Extreme Value Theory and its Applications," John Wiley & Sons, Inc., Hoboken, New Jersey.
- Olivares, A., and Tompkins, M. (2016) "Clearing and Settlement Systems from Around the World: A Qualitative Analysis," Bank of Canada Staff Discussion Paper No. 2016-14.
- Pérez Saiz, H. and Xerri, G. (2016). "Credit Risk and Collateral Demand in a Retail Payment System," Discussion Papers 16-16, Bank of Canada.
- Perez-Saiz, H., Williams, B., and Xerri, G. (2018). "Tail Risk in a Retail Payment System: An Extreme-Value Approach," Discussion Papers 18-2, Bank of Canada.
- Roodman, David. 2015. "EXTREME: Stata module to fit models used in univariate extreme value theory," *Statistical Software Components S457953*, Boston College Department of Economics, revised 18 Dec 2017.

Tables and Figures

Table 1 – Select GPD and GEV Model Estimates

	GPD		GEV	
	\$800M Threshold	Daily	Weekly	Monthly
μ		2.696e+08*** (2604944)	5.010e+08*** (8749894)	8.253e+08*** (2.2E+07)
$\ln(\sigma)$	19.439*** (0.094)	18.788*** (0.014)	19.132*** (0.033)	19.438*** (0.058)
ξ	-0.064 (0.065)	0.186*** (0.014)	0.116*** (0.031)	-0.040 (0.052)
N	212	4029	706	192
log likelihood	-4,319.47	-82,485.4	-14,669.9	-4,030.78
		Cox-Snell bias correction		
ξ	-0.048 (0.057)	0.186*** (0.012)	0.117*** (0.028)	-0.037 (0.046)
		Estimated Return Levels (in billions)		
10 year	2.00	2.82	2.40	2.03
95% CI	[1.75, 2.25]	[2.53, 3.10]	[2.00, 2.80]	[1.77, 2.29]

Note: *** denotes statistical significance at the 99% level. Standard errors reported in parentheses. 1,000 bootstrap replications are used for the Cox-Snell bias corrected standard error estimates. Our estimation sample is derived from ACSS data over the time period January 2002 to March 2017.

Figure 1 – Value and Volume of Payments

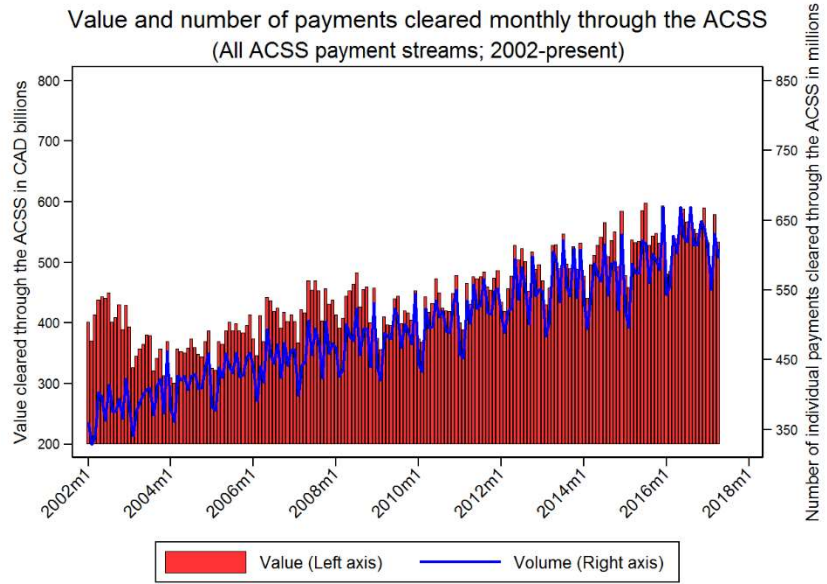
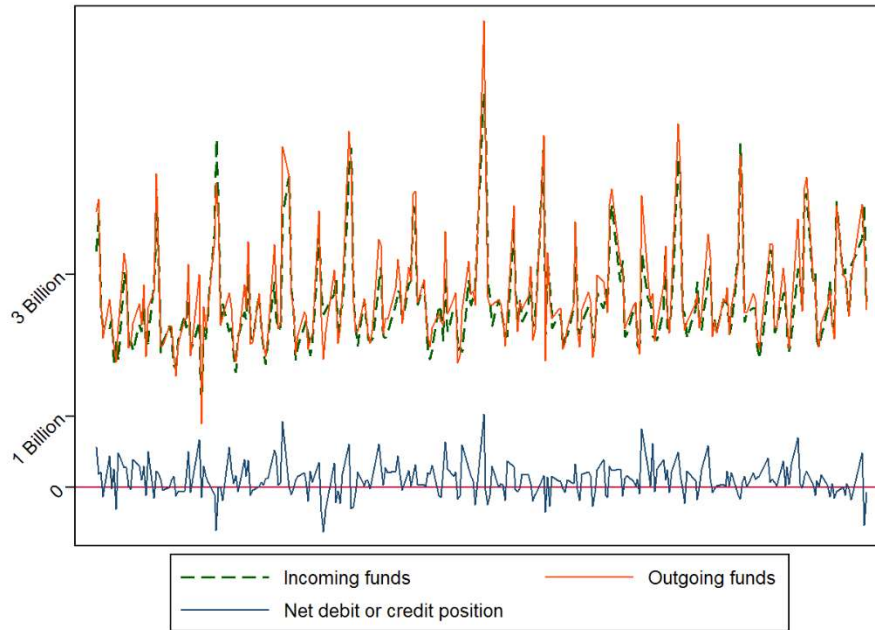
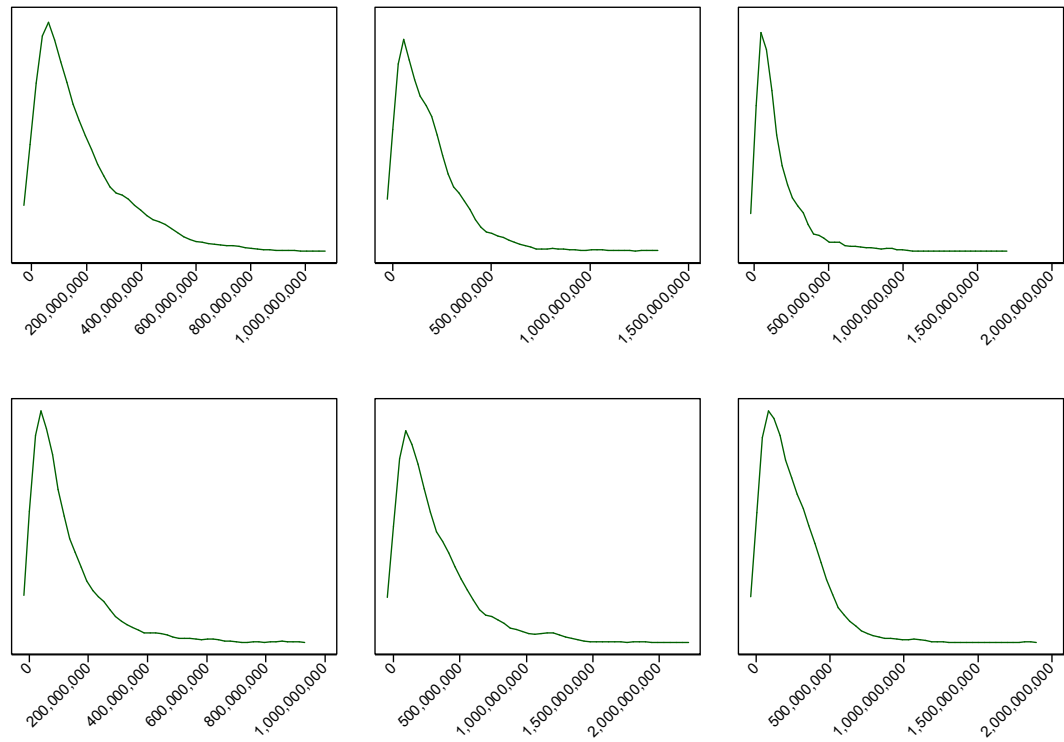


Figure 2 – Visualizing Payment Flows for a Select Participant



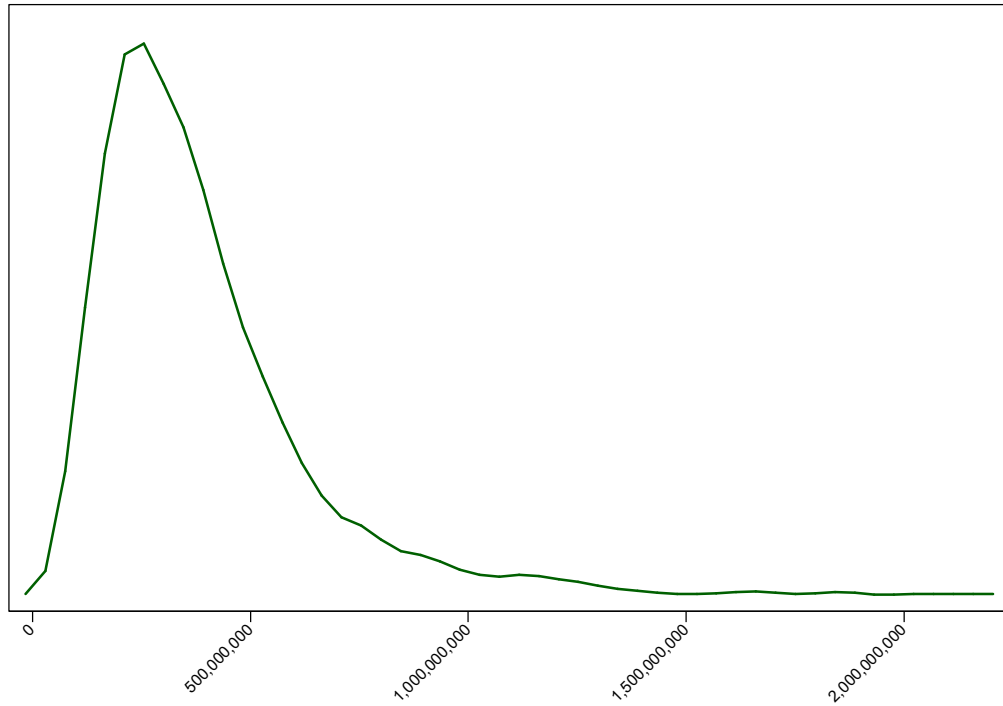
Note: Y-axis in dollars and arbitrarily re-scaled. X-axis measures time in days, intentionally left blank.

Figure 3 – Density Plots of End of Day Net Debit Positions for Select Participants



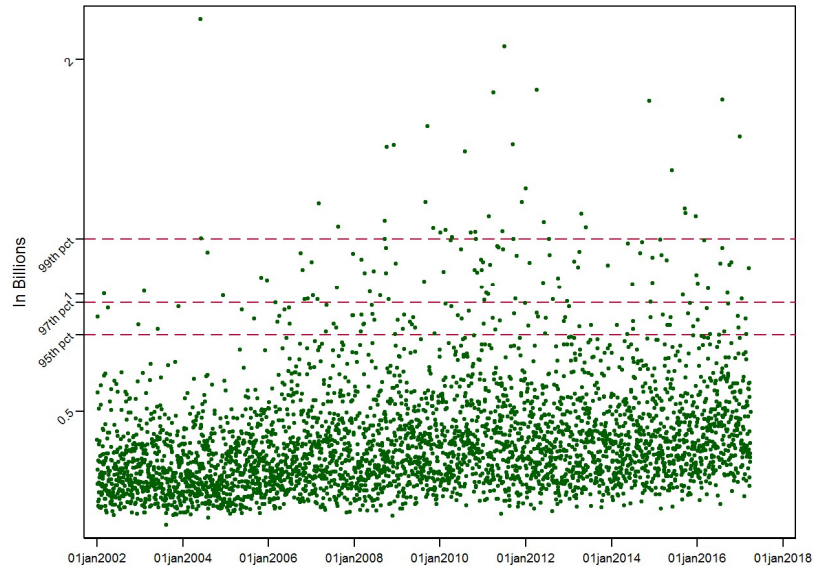
Note: The figures show kernel density plots using daily end of day net debit position data for select participants over the time period January 2002 to March 2017. X-axis denotes dollars; y-axis denotes density.

Figure 4 – Daily Maximum Net Debit Position across ACSS Participants



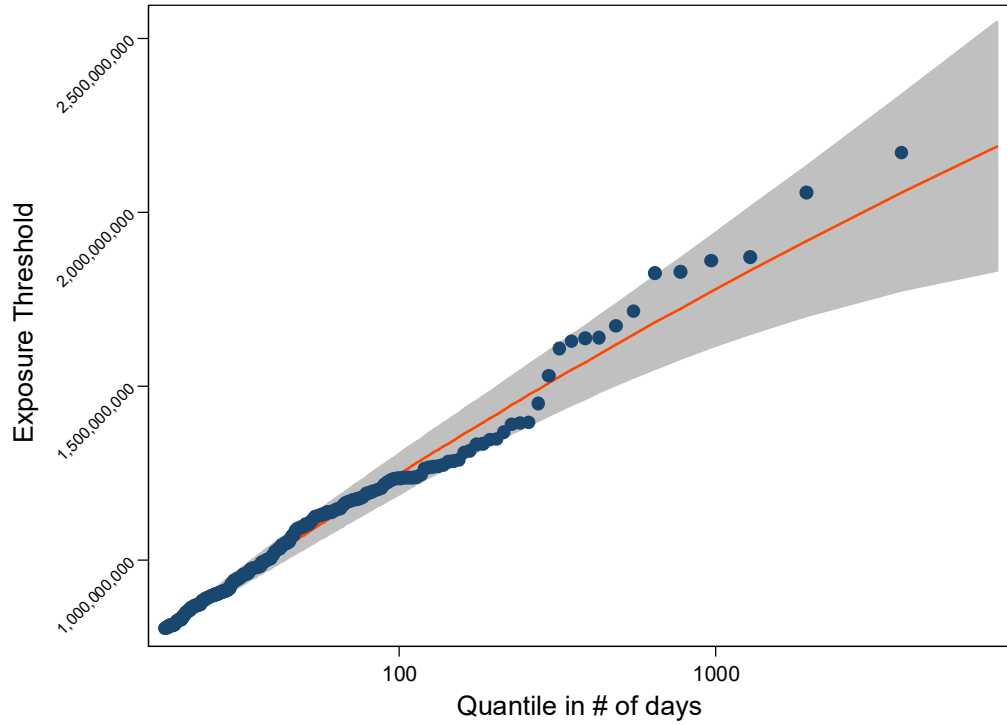
Note: Kernel density plot of the daily maximum end of day net debit position across participants over the time period January 2002 to March 2017. X-axis denotes dollars; y-axis denotes density.

Figure 5 – Historical Maximum Daily Net Debit Position in ACSS and Empirical Quantiles



Note: Scatter points measure the largest net debit position in ACSS observed across all DCs for each business day over the time period January 2002 to March 2017. Empirical quantiles computed ex post and 99th percentile corresponds to a threshold amount that is exceeded on only one percent of days.

Figure 6 – Risk Curve: Return Period in Days – GPD Model



Note: Estimation sample is based on the GPD model in Table 1. Model estimate (red line) and confidence interval shaded in grey are overlapped with empirical quantiles (blue data points). Data is fit to a peak-over-threshold Generalized Pareto distribution using maximum likelihood. Estimated shape parameter indicates an upper limit on extreme values and a concave risk curve.

Figure 7 – GPD Model Diagnostic Plot – Empirical vs Modeled Quantile

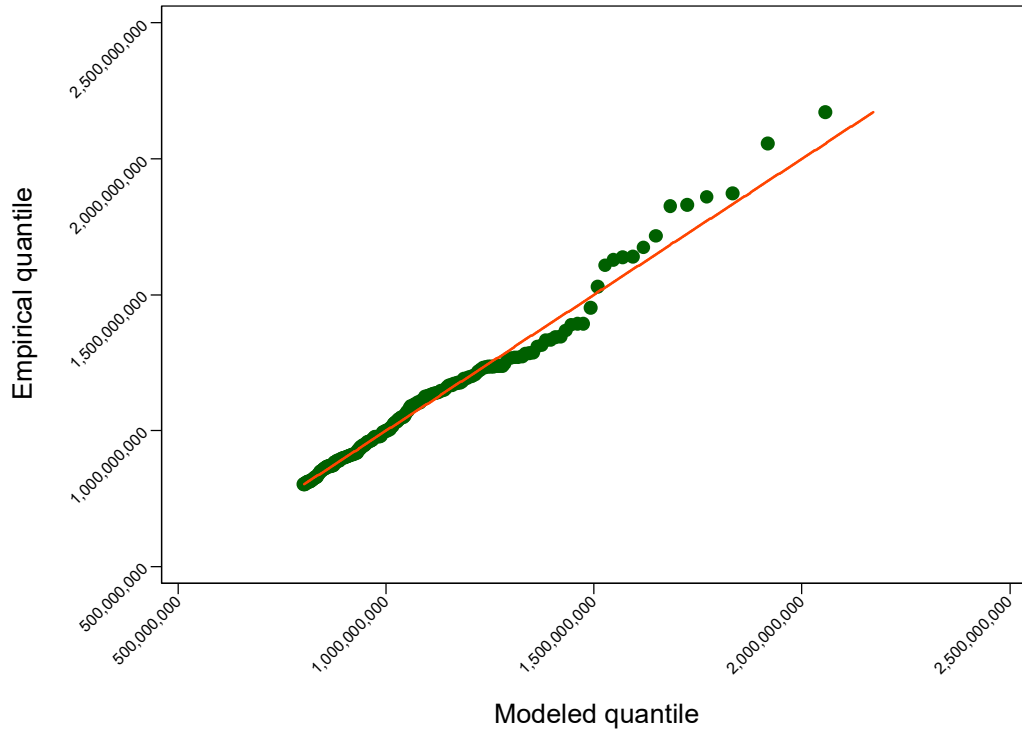


Figure 8 – GPD Diagnostic – Varying Shape Parameter and Threshold Level

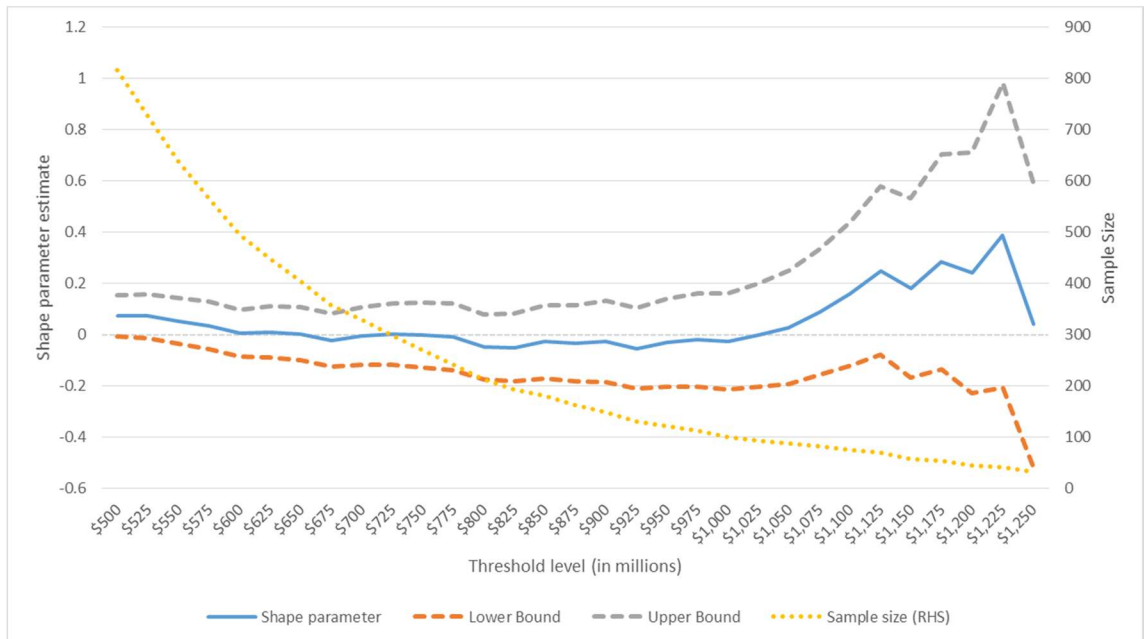


Figure 9 – Sensitivity Analysis, Varying 10 year return levels by Shape Parameter

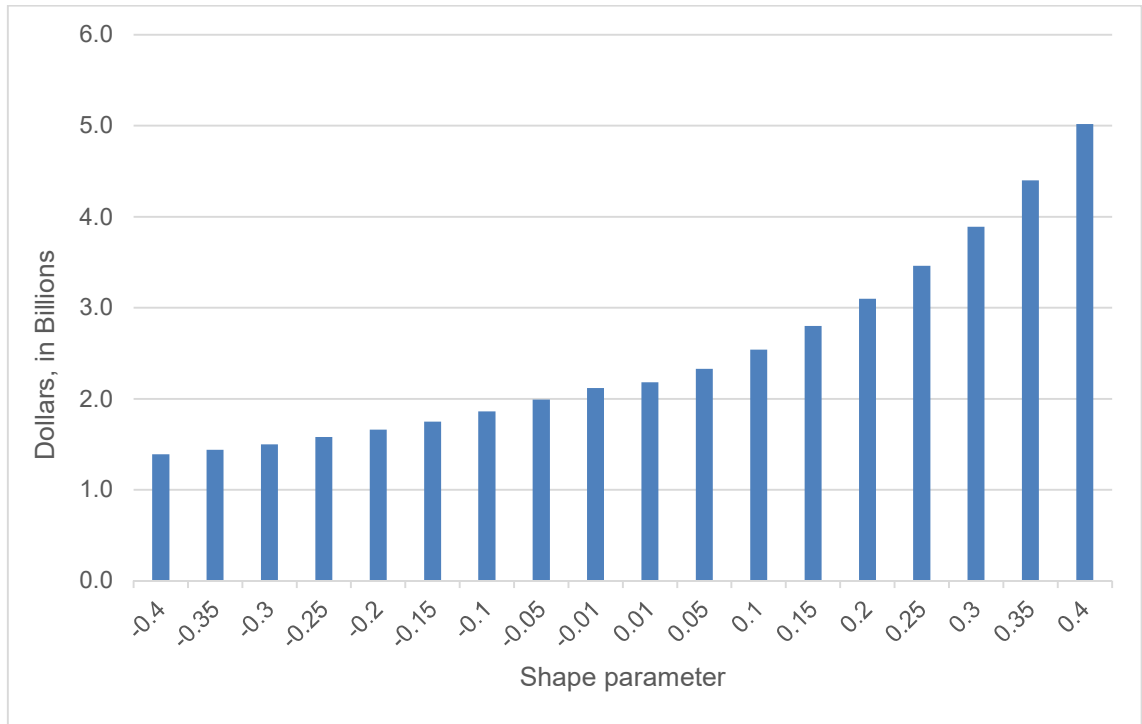
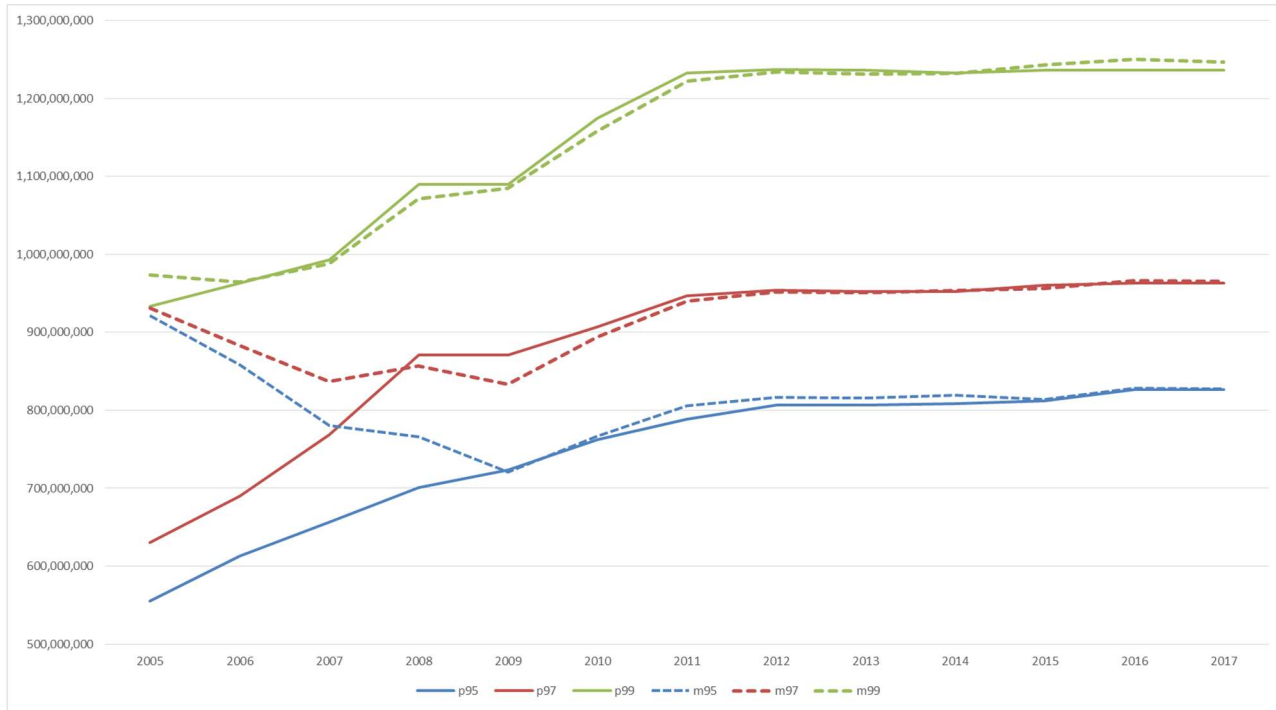


Figure 10 – Evolution of Empirical and Model Quantiles as the Historical Sample Size Increases



Note: 2005 model and empirical estimates are based on estimation sample covering January 2002 to December 2005. For subsequent years, model and empirical estimates are re-updated by including additional observations for the respective year to the estimation sample. p95, p97, p99 correspond to empirical quantiles; m95, m97, m99 correspond to model-based quantiles.

Figure 11 – Rolling Cover-One Collateral Pool, Updated Monthly (GPD monthly window model)

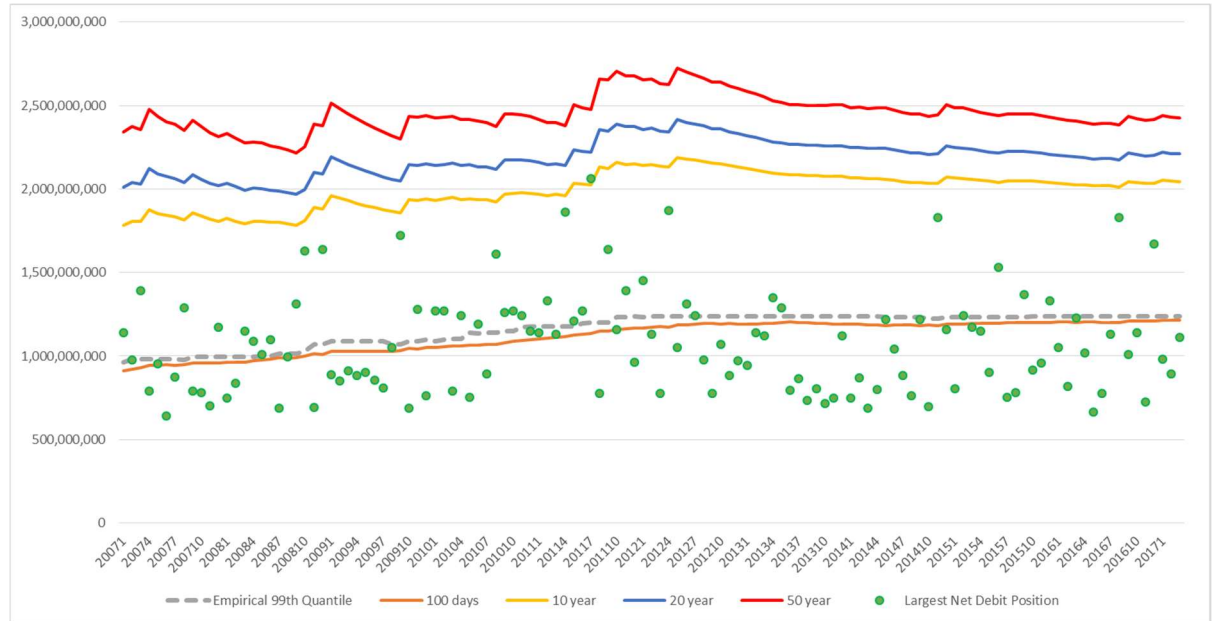
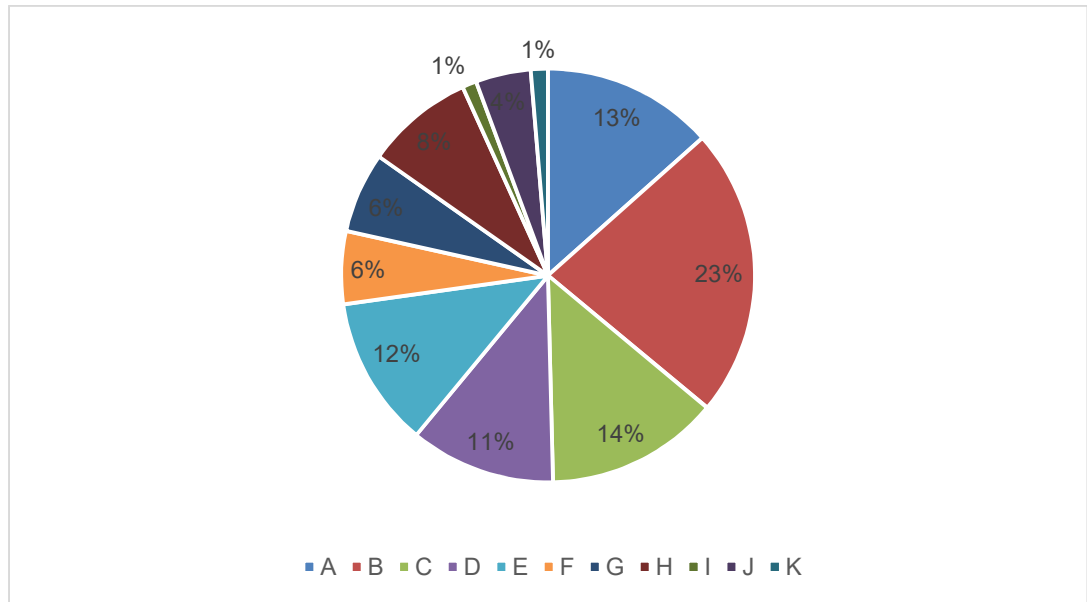


Figure 12 – Collateral Pool Allocation Based on Proportional Size of 10 year model return periods



Note: A-K represent individual participant percentage contributions to a collateral pool. Participant B has the largest contribution to a collateral pool.

Figure 13 – Participant Contributions to a rolling collateral pool (GEV monthly window model)

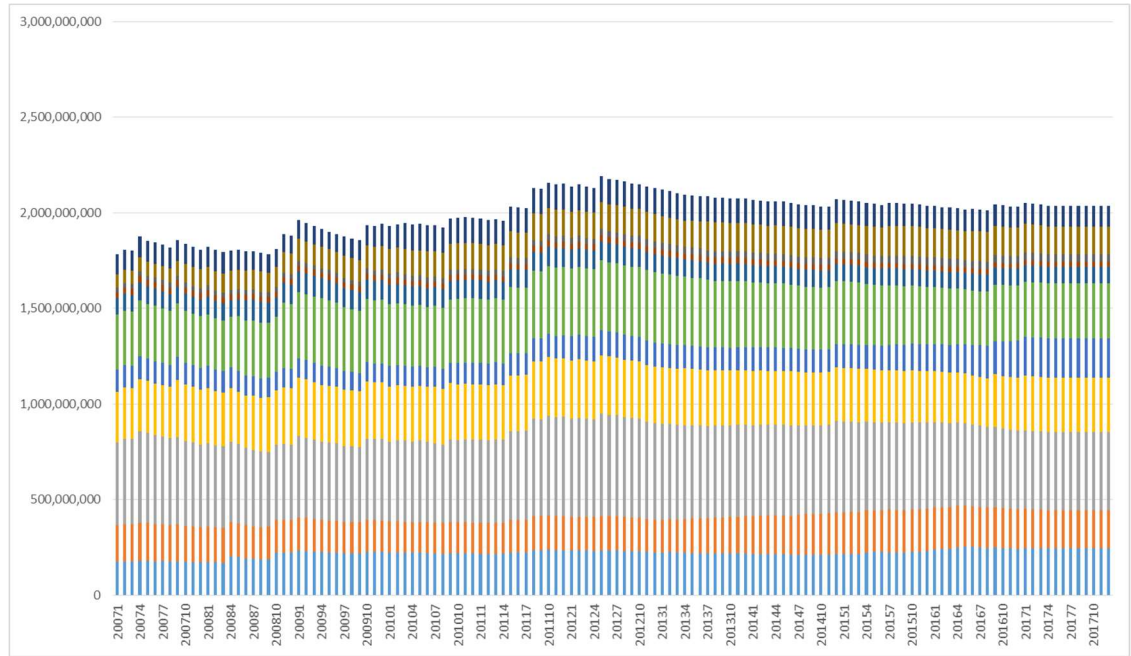
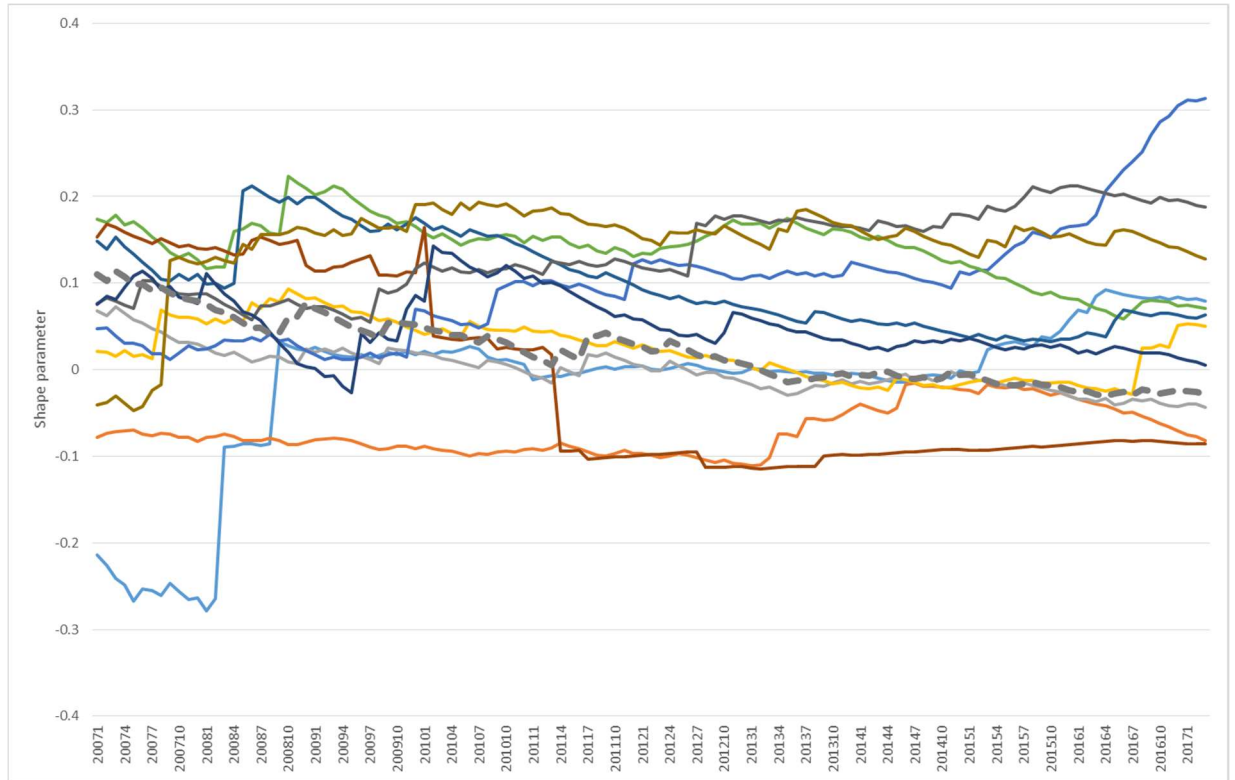


Figure 14 – Individual Participant Rolling Shape Parameter Estimates (GEV Monthly window)



Note: Dashed grey line represent shape parameter estimates using the series of daily maximum net debit positions in determination of aggregate exposure for the case of cover-one.